Lecture course Riemannian Geometry II (SP1 Z2):

Lecturer: Thomas Schick

Lectures: Tuesday and Friday 10-12 (planned). 6 credit points, the course will be designed as 3+1 or 2+2 (lecture/exercises)

Assitance: Henry Fischer

Language: English.

Audience: Participants of Riem. Geom. 1. The module works well for M.Sc. students, for B.Sc. students a creative solution still has not been found.

Exam requirements: active participation in the course (no strict formal requirements)

Modules: M.Mat.4613: Aspects of differential geometry; M.Mat.4624: Aspects of groups, geometry and dynamical systems; M.Mat.4615: Aspects of mathematical methods in physics

Description

The course is the continuation of Riemannian geometry I and will built on the latter material. Entering now is possible if the prerequisit knowledge has been obtained in a different way (course taken earlier/elsewhere, independent study,..).

The course will explore more of the relations between geometry, in particular curvature, and topology. We will learn about curvature of submanifolds. In particular, we will compare extrinsic and intrinsic curvature and learn the modern version of Gauss' "Theorema Egregium". We will also study the intrinsic Riemannian notion of volume and the relation between curvature and volume (Bishop-Gromov inequality). If time permits, we will also cover some aspects of spectral theory of the Laplace operator on a Riemannian manifold and topological consequences. Or we might venture into the concept of "convergence and degeneration" of sequences of Riemannian manifolds and how this can be used to classify and control the set of all (or many) of them.

As goal of the course, the participants will know:

- how a Riemannian metric gives rise to an intrinsic notion of volume and integraion
- how the volume is influenced by curvature (and vice versa)
- how one distinguishes intrinsic "curvedness" of a Riemannian manifold from extrinsic "being embedded in a curved way" into another manifold (and Gausses revelation that the two notions are related)
- other aspects of how the geometry influences the manifold

Literature

[1] Marcel Berger, *A panoramic view of Riemannian geometry*, Springer-Verlag, Berlin, 2003. MR2002701

[2] Arthur L. Besse, *Einstein manifolds*, Classics in Mathematics, Springer-Verlag, Berlin, 2008. Reprint of the 1987 edition. MR2371700

[3] William M. Boothby, *An introduction to differentiable manifolds and Riemannian geometry*, 2nd ed., Pure and Applied Mathematics, vol. 120, Academic Press, Inc., Orlando, FL, 1986. MR861409

[4] Isaac Chavel, *Riemannian geometry*, 2nd ed., Cambridge Studies in Advanced Mathematics, vol. 98, Cambridge University Press, Cambridge, 2006. A modern introduction. MR2229062

[5] Manfredo Perdigão do Carmo, *Riemannian geometry*, Mathematics: Theory & Applications, Birkhäuser Boston, Inc., Boston, MA, 1992. Translated from the second Portuguese edition by Francis Flaherty. MR1138207

[6] Sylvestre Gallot, Dominique Hulin, and Jacques Lafontaine, *Riemannian geometry*, 3rd ed., Universitext, Springer-Verlag, Berlin, 2004. MR2088027

[7] Jürgen Jost, *Riemannian geometry and geometric analysis*, 7th ed., Universitext, Springer, Cham, 2017. MR3726907

[8] John M. Lee, *Introduction to Riemannian manifolds*, Graduate Texts in Mathematics, vol. 176, Springer, Cham, 2018. Second edition of [MR1468735]. MR3887684

[9] Peter Petersen, *Riemannian geometry*, 3rd ed., Graduate Texts in Mathematics, vol. 171, Springer, Cham, 2016. MR3469435

[10] Takashi Sakai, *Riemannian geometry*, Translations of Mathematical Monographs, vol. 149, American Mathematical Society, Providence, RI, 1996. Translated from the 1992 Japanese original by the author. MR1390760